# D-brane instanton effects in Type II orientifolds: local and global issues

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We review how D-brane instantons can generate open string couplings of stringy hierarchy in the superpotential which violate global abelian symmetries and are therefore perturbatively forbidden. We discuss the main ingredients of this mechanism, focussing for concreteness on Euclidean $D2$-branes in Type IIA orientifold compactifications. Special emphasis is put on a careful analysis of instanton zero modes and a classification of situations leading to superpotential or higher fermionic F-terms. This includes the discussion of chiral and non-chiral instanton recombination, viewed as a multi-instanton effect. As phenomenological applications we discuss the generation of perturbatively forbidden Yukawa couplings in SU(5) GUT models and Majorana masses for right-handed neutrinos. Finally we analyse the mirror dual description of $D1$-instantons in Type I compactifications with $D9$-branes and stable holomorphic bundles. We present globally defined semi-realistic string vacua on an elliptically fibered Calabi-Yau realising the non-perturbative generation of Majorana masses.

## 1 Introduction

During the last year there has been some progress towards a better understanding of non-perturbative effects in supersymmetric four-dimensional string compactifications on Calabi-Yau orientifolds. As was realized in [1, 2, 3, 4], D-brane instantons can induce couplings between open string fields which are perturbatively forbidden because they violate global $U(1)$ selection rules. These effects are intrinsically stringy in that they cannot be described by conventional gauge instantons. For Type IIA orientifolds with intersecting D6-branes the relevant class of instantons is given by Euclidean $D2$-brane instantons, short $E2$-instantons, wrapping special Lagrangian three-cycles of the internal Calabi-Yau space [1, 3].

In general the gauge group $U(N_a)$, carried by $N_a$ coincident D-branes, contains an anomalous $U(1)_a$ which becomes massive via the generalized Green-Schwarz mechanism and survives as a global perturbative symmetry. It is due to this $U(1)$ selection rule that particular phenomenologically important couplings are absent in intersecting brane worlds. These include Majorana masses for the right-handed neutrino, $\mu$-terms for the MSSM Higgs sector or Yukawa-couplings of type $\mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}_H$ in $SU(5)$-like GUT models. On the other hand, in the dual strongly coupled description in terms of M-theory compactified on $G_2$-manifolds the $U(1)_a$ decouples completely. There is therefore no associated selection rule and no obstruction for the above mentioned couplings to exist. The resolution to this puzzle is given by $U(1)_a$ breaking non-perturbative terms in the type IIA picture.

Indeed, from the axionic shift symmetries under the abelian symmetries induced by the Chern-Simons couplings of the $N_a$ $D6_a$-branes one finds the $U(1)_a$ transformation of the instanton [1, 3]

$$e^{-S_{E2}} = \exp\left[\frac{2\pi}{\ell_s^3}\left(-\frac{1}{g_s}\text{Vol}_\Xi + i\int_\Xi C^{(3)}\right)\right] \longrightarrow e^{i\,Q_a(E2)\,\Lambda_a}\;e^{-S_{E2}} \tag{1}$$

with

$$Q_a(E2) = N_a \, \Xi \circ (\Pi_a - \Pi'_a). \tag{2}$$

Thus the exponential suppression factor $e^{-S_{E2}}$ characteristic for instantonic couplings transforms under the global $U(1)$ symmetries in such a way that the full coupling

$$W_{np} = \prod_i \Phi_{a_i b_i} \, e^{-S_{E2}} \tag{3}$$

is invariant again. In this way an instanton with the appropriate zero mode structure has the potential to generate perturbatively forbidden couplings.

Various applications of the associated effects in different branches of the string landscape have recently appeared in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29], with related earlier work including [30, 31, 32].

In the first part of this article we review some of the technical aspects in dealing with stringy D-brane instantons, based mainly on [1, 8, 17]. In section 2 we give a detailed account of the zero mode structure of D-brane instantons in Type II orientifolds, focusing for concreteness on $E2$-instantons in Type IIA. In section 3 we systematically distinguish between various situations which lead either to genuine superpotential contributions or to higher fermionic F-terms as discussed originally by Beasley and Witten in the context of heterotic worldsheet instantons. A novelty for half-BPS instantons in Type II orientifolds (as opposed to heterotic theories) is the appearance of four instead of two Goldstone fermions. In section 4 we discuss how these modes can be lifted by couplings in the effective instanton action which effectively describes the configuration as a two-instanton process in the upstairs geometry before orientifolding [17]. Unless further zero modes such as certain charged or deformation modes remain unlifted, this shows that the presence of these Goldstinos does not necessarily obstruct the generation of a superpotential. We then exemplify, in section 5, some phenomenological applications of stringy instantons by detailing the generation of the aforementioned Yukawa couplings in the context of SU(5) GUT models [15] and the generation of Majorana mass terms [1, 3]. The construction of globally consistent string vacua exhibiting these effects turns out to be easier in the mirror dual language of Type I compactifications, which are the subject of section 6. We apply the general setting to the construction of a class of semi-realistic GUT type vacua on elliptically fibered manifolds where the instanton contribution to Majorana masses can indeed be proven to be non-vanishing [24].

## 2   Instanton zero modes

The computation of D-brane instanton effects hinges upon a careful analysis of the instanton zero modes. For definiteness we focus now on compactifications of Type IIA on Calabi-Yau orientifolds with intersecting D6-branes (see [33, 34, 35, 36, 37] for reviews). The relevant spacetime instantons are given by $E2$-branes wrapping special Lagrangian three-cycles $\Xi$ in the Calabi–Yau so that they are point-like in four-dimensional spacetime. In section 6 of this article we describe the mirror symmetric picture of $E1$-instantons in Type I compactifications.

One distinguishes between two kinds of instanton zero modes corresponding to whether or not they are charged under the gauge groups on the $D6$-branes. The uncharged zero modes arise from the $E2$-$E2$ sector. They always comprise the universal four bosonic Goldstone zero modes $x^\mu$ due to the breakdown of four-dimensional Poincaré invariance. Generically, for instantons away from the orientifold fixed plane, these come with four fermionic zero modes $\theta^\alpha$ and $\overline{\tau}^{\dot\alpha}$ [9, 10, 11, 12]. This reflects the fact that the instanton breaks half of the eight supercharges preserved by the Calabi-Yau manifold away from the orientifold fixed plane. The $\theta^\alpha$ modes indicate the breakdown of one half of the $\mathcal{N} = 1$ supersymmetry preserved by the orientifold while the $\overline{\tau}^{\dot\alpha}$ modes are associated with the breakdown of one half of its orthogonal complement inside the $\mathcal{N} = 2$ supersymmetry algebra respected by the internal Calabi-Yau. For instantons on top of an orientifold plane these universal zero modes are subject to the orientifold projection. For effective

$O6^-$-planes[1], which we always assume in the sequel, the non-dynamical gauge group of $N$ coincident instantons is $O(N)$ and $x^\mu, \theta^\alpha$ are symmetrized, while the anti-chiral modes $\overline{\theta}_{\dot\alpha}$ are anti-symmetrized. Thus for a single $O(1)$ instanton the zero modes surviving the orientifold action are $x^\mu$ and $\theta^\alpha$.

Generically the $E2 - E2$ sector exhibits $b_1(\Xi)$ complex bosonic zero modes $c_I$ associated with special Lagranigan deformations of the $E2$-instanton. Away from the orientifold plane, each of these is accompanied by one chiral and one anti-chiral Weyl spinor, $\chi_I^\alpha$ and $\overline{\chi}_I^{\dot\alpha}$. For an instanton wrapping an $\Omega\overline{\sigma}$ invariant cycle $c_I, \overline{c}_I$ and $\overline{\chi}_I^{\dot\alpha}$ are symmetrized while $\chi_I^\alpha$ are anti-symmetrized or vice versa, depending on the type of cycle the instanton wraps. In the sequel we refer to them as deformations of the first and second kind, respectively. In the T-dual Type I picture they correspond to the position and Wilson line moduli of an $E1$-instanton wrapping a holomorphic curve.

Furthermore there arise zero modes at non-trivial intersections of the instanton $E2$ with its orientifold image $E2'$. Intersections away from the orientifold give rise to a chiral supermultiplet $(m, \mu^\alpha)$ and its anti-chiral counterpart. For $\Omega\overline{\sigma}$ invariant intersections the states are subject to the additional orientifold projection which symmetrizes the zero modes $m, \overline{m}, \overline{\mu}^{\dot\alpha}$ and anti-symmetrizes the chiral mode $\mu^\alpha$. Thus for a single $U(1)$-instanton we get the multiplicities for the zero modes as shown in table 1.

| zero mode | $(Q_E)_{Q_{ws}}$ | Multiplicity |
|-----------|------------------|--------------|
| $m, \overline{m}$ | $(2)_1$ ,$(-2)_{-1}$ | $\frac{1}{2}\left(\Xi' \circ \Xi + \Pi_{O6} \circ \Xi\right)$ |
| $\overline{\mu}^{\dot\alpha}$ | $(-2)_{1/2}$ | $\frac{1}{2}\left(\Xi' \circ \Xi + \Pi_{O6} \circ \Xi\right)$ |
| $\mu^\alpha$ | $(2)_{-1/2}$ | $\frac{1}{2}\left(\Xi' \circ \Xi - \Pi_{O6} \circ \Xi\right)$ |

**Table 1** Charged zero modes on $E2 - E2'$ intersection.

This completes our discussion of the uncharged zero mode sector. In addition, there can arise fermionic zero modes from intersections of the instanton $\Xi$ with $D6$-branes $\Pi_a$ [1, 3, 4]. Let us focus for brevity on chiral intersections. As pointed out in [8] states in the $E2 - D6$ sector are odd under the GSO projection contrary to the GSO-even states in the $D6 - D6$ brane sector. In particular, a positive intersection $I_{\Xi a} > 0$ of the instanton and a D6-brane wrapping the respective cycles $\Xi$ and $\Pi_a$ hosts a single *chiral* fermion (i.e. with world-sheet charge $Q_{ws} = -\frac{1}{2}$) in the bifundamental representation $(-1_E, \square_a)$. The strict chirality of the charged fermions is essential for the existence of holomorphic couplings between these modes and open string states in the moduli action and will also play a key role in the present analysis. For a generic instanton cycle $\Xi$ away from the orientifold, this gives rise to the charged zero mode spectrum summarised in table 2. As a result, the instanton carries the charge $Q_a(E2) = \mathcal{N}_a\, \Xi \circ (\Pi_a - \Pi_a')$ under the gauge

| zero modes | $\text{Reps}_{Q_{ws}}$ | number |
|------------|------------------------|--------|
| $\lambda_{a,I}$ | $(-1_E, \square_a)_{-1/2}$ | $I = 1, \ldots, [\Xi \cap \Pi_a]^+$ |
| $\overline{\lambda}_{a,I}$ | $(1_E, \overline{\square}_a)_{-1/2}$ | $I = 1, \ldots, [\Xi \cap \Pi_a]^-$ |
| $\lambda_{a',I}$ | $(-1_E, \overline{\square}_a)_{-1/2}$ | $I = 1, \ldots, [\Xi \cap \Pi_a']^+$ |
| $\overline{\lambda}_{a',I}$ | $(1_E, \square_a)_{-1/2}$ | $I = 1, \ldots, [\Xi \cap \Pi_a']^-$ |

**Table 2** Zero modes at chiral $E2$ -$D6$ intersections.

group $U(1)_a$ [1, 3].

## 3 Instanton generated F-terms

All bosonic and fermionic zero modes appear in the instanton measure. The instanton induced amplitude is only non-vanishing if all fermionic zero modes can be soaked up. In this section we analyse the possible

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[1] In contrast to effective $O6^-$-planes effective $O6^+$-planes admit no supersymmetric tadpole cancellation in $D6$-brane models and are thus not of phenomenological interest.

instanton generated F-term contributions, taking into account systematically which of the above zero modes are present in each case.

### 3.1    Rigid $O(1)$ instantons

We start with an $O(1)$ instanton wrapping a rigid cycle in the internal Calabi-Yau. It therefore contains only the four universal bosonic zero modes $x^\mu$ and the two chiral fermionic zero modes $\theta^\alpha$ in the uncharged sector. If in addition all charged zero modes arising at intersections with $D6$-branes can be soaked up via disk diagrams[2] $< \lambda^a_{-1/2} \Phi^{ab}_1 \overline{\lambda}^b_{-1/2} >$, where $\Phi_{ab} = \phi_{ab} + \theta\psi_{ab}$ denotes the chiral superfield arising at the intersection of branes $a$ and $b$, such an instanton induces superpotential terms of the form

$$W \simeq \prod_{i=1}^{M} \Phi_{a_i,b_i}\, e^{-S_{E2}}. \tag{4}$$

Note that in case the product $\prod_{i=1}^{M} \Phi_{a_i,b_i}$ is not invariant under the massive $U(1)_i$ such a coupling is perturbatively forbidden and the instanton induced F-term represents the leading contribution rather than just a correction to the effective action.

For a systematic treatment of the associated instanton calculus in this context we refer to [1] and for its concrete application in a toroidal orbifold to [8]. Due to the peculiar $\sqrt{g_s}$-scaling of the vertex operators of the charged zero modes, only disk diagrams containing precisely two $\lambda$-modes can contribute to the superpotential, while one-loop amplitudes with one end on the instanton and one end on the $D6$-branes do not carry any instantonic charged zero modes. Furthermore, the classical instanton suppression $S_{E2} = \frac{2\pi}{\ell_s^3}(-\frac{1}{g_s}\mathrm{Vol}_\Xi + i\int_\Xi C^{(3)})$ receives corrections from one-loop amplitudes with no operator insertions [1]. These yield [5, 6] the threshold corrections [38, 39] to the hypothetical gauge coupling associated with the cycle $\Xi$ and ensure holomorphicity of the superpotential [13, 20, 40].

### 3.2    Non-rigid $O(1)$ instantons: Superpotential and higher fermionic terms

Let us turn to an $O(1)$ instanton which does not wrap a rigid cycle and thus carries additional deformation modes. While deformation modes of the second kind induce corrections to the gauge kinetic function [13] we focus on deformation modes of the first type. Generically, they generate higher fermionic couplings for the closed string fields [41]. Before discussing these couplings let us examine under what circumstances such an instanton could contribute instead to the superpotential [17]. For simplicity we assume that $b_1(\Xi) = 1$. The orientifold action projects out the chiral modes $\chi^\alpha$ and the extra surviving non-charged zero modes are $c, \overline{c}$ and $\overline{\chi}^{\dot\alpha}$. For superpotential contributions to exist it must be possible to absorb the fermionic zero modes $\overline{\chi}^{\dot\alpha}$ without generating higher fermionic or derivative terms. A way to do this for matter field superpotential contributions is via a five-point amplitude

$$< \overline{\chi}^{\dot\alpha}_{1/2}\, \overline{\chi}^{\dot\alpha}_{1/2}\, \lambda^a_{-1/2}\, \Phi^{ab}_0\, \overline{\lambda}^b_{-1/2} > . \tag{5}$$

If this five-point function has a contact term and if the remaining integral over the bosonic instanton moduli space does not vanish, a contribution to the superpotential can be generated. Let us stress that from a general $\mathcal{N} = 2$ SCFT point of view, no obvious selection rules forbid such an interaction term. Having said this, one can easily convince oneself that e.g. for factorisable three-cycles on toroidal orbifolds the amplitude vanishes due to violation of the $U(1)$ worldsheet charge which has to be conserved for each of the three tori separately. This, however, need not be so for more general setups and has to be determined in each case.

As observed in [41] in the S-dual heterotic context such an instanton (and its anti-instanton) can also

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[2] Here and in the sequel the subscripts denote the worldsheet $U(1)$-charges. Disks containing more than one matter superfields are likewise possible.

generate higher fermionic F-terms which are encapsulated in interactions of the form

$$S = \int d^4x \, d^2\theta \; w_{\overline{ij}}(\Phi) \overline{\mathcal{D}}^{\dot{\alpha}} \overline{\Phi}^{\overline{i}} \overline{\mathcal{D}}_{\dot{\alpha}} \overline{\Phi}^{\overline{j}} + h.c. \tag{6}$$

for the simplest case that the instanton moves in a one-dimensional moduli space. Note that supersymmetry requires a holomorphic dependence of $w_{\overline{ij}}(\Phi)$ on the superfields $\Phi$.

First let us reproduce the result associated with a deformation of the closed string moduli space obtained by Beasley and Witten in [41] in the present context [17]. Thus we assume no additional charged instantonic zero modes. Denoting by $\mathcal{T} = T + \theta^{\alpha} t_{\alpha}$ the $\mathcal{N} = 1$ chiral superfield associated with the Kähler moduli, we can absorb the instanton modulini by pulling down from the moduli action two copies of the schematic anti-holomorphic coupling $\overline{\chi}^{\dot{\alpha}} \overline{t}_{\dot{\alpha}}$. In general the open-closed amplitude $\langle \overline{\chi}^{\dot{\alpha}} \overline{t}_{\dot{\alpha}} \rangle$ does not violate any obvious selection rule of the $\mathcal{N} = (2,2)$ worldsheet theory and is therefore expected to induce the above coupling. Similarly, the two $\theta$-modes can be soaked up by the holomorphic coupling $\theta^{\alpha} u_{\alpha}$ involving the fermionic partners of the complex structure moduli encoded in the superfield $\mathcal{U} = U + \theta^{\alpha} u_{\alpha}$. This results in a four-fermion interaction of the schematic form $e^{-S_{E2}} uu \overline{tt}$. Note that the coupling of the complex and Kähler structure modulini only to the universal and reparametrisation zero modes, respectively, is a consequence of $U(1)$ worldsheet charges of the associated vertex operators. The derivative superpartner to the above four-fermi term is obtained by integrating out two copies of the term $\theta \sigma^{\mu} \overline{\chi} \, \partial_{\mu} \overline{T}$ in the instanton effective action, whose presence follows from evaluating the amplitude $\langle \theta^{\alpha} \, \overline{\chi}^{\dot{\alpha}} \, \overline{T} \rangle$. This can be summarized in superspace notation by writing

$$S = \int d^4x \, d^2\theta \; e^{-\mathcal{U}(\Xi)} \, f_{\overline{i}, \overline{j}} \left( e^{-\mathcal{T}_i}, e^{-\Delta_i} \right) \overline{\mathcal{D}}^{\dot{\alpha}} \overline{\mathcal{T}}^{\overline{i}} \overline{\mathcal{D}}_{\dot{\alpha}} \overline{\mathcal{T}}^{\overline{j}}, \tag{7}$$

where $\mathcal{U}(\Xi)$ is associated with the specific combination of complex structure moduli appearing in the classial instanton action and the holomorphic function $f_{\overline{i}, \overline{j}}$ depends in general on the Kähler and open string moduli of the $D6$-branes $\Delta_i$. This is indeed of the form (6) proposed in [41].

In the presence of a suitable number of charged $\lambda$ zero-modes there exist, in addition to these closed string couplings, terms which generate higher fermi-couplings also for the matter fields. The Chan-Paton factors and worldsheet selection rules only allow the $\lambda$ modes to couple holomorphically to the chiral open string superfields, as for the generation of a superpotential (5), such that the instanton induces an interaction as in (7), but with $e^{-\mathcal{U}(\Xi)}$ simply replaced by $e^{-\mathcal{U}(\Xi)} \prod_{a_i, b_i} \Phi_{a_i, b_i}$ (and modified coupling $f_{\overline{i}, \overline{j}}$).

For configurations with the correct charged zero mode structure, the action can also pick up derivative terms directly involving the open string fields. For this to happen the instanton moduli action has to contain couplings of the form $\overline{\chi}_{1/2} \lambda^a_{-1/2} \, \overline{\psi}^{ab}_{1/2} \, \overline{\lambda}^b_{-1/2}$, where the fermionic matter field $\overline{\psi}^{\dot{\alpha}}_{1/2}$ lives at the intersection $D6_a - D6_b$ and lies in the anti-chiral superfield $\overline{\Phi}$.

Integrating out two copies of this interaction term brings down the fermion bilinear $\overline{\psi}_{1/2} \overline{\psi}_{1/2}$. In addition, the two $\theta^{\alpha}$ modes again pull down a bilinear of *chiral* fermions $u^{\alpha}$ or, in the presence of more $\lambda$ modes, $\psi^{\alpha}_{ab}$, as in the case of superpotential contributions. This induces again a four-fermi coupling. Alternatively, we can absorb one pair of $\theta^{\alpha} \overline{\chi}^{\dot{\alpha}}$ via a disk amplitude of the form $< \theta_{3/2} \, \overline{\chi}_{1/2} \, \lambda^a_{-1/2} \, \overline{\phi}_{-1} \, \overline{\lambda}^b_{-1/2} >$. After bringing the $\overline{\phi}_{-1}$ into the zero ghost picture this clearly generates a derivative coupling of the form $\theta \sigma^{\mu} \overline{\chi} \, \lambda^a \, \partial_{\mu} \overline{\phi} \, \overline{\lambda}^b$. Integrating out two copies of this term yields the derivative superpartner to the above four-fermi term.

### 3.3 Rigid $U(1)$-instantons not intersecting the O-plane

Finally let us discuss a rigid $U(1)$ instanton which does not intersect its orientifold image. This instanton contains all the universal zero modes $x^{\mu}, \theta^{\alpha}$ and $\overline{\tau}^{\dot{\alpha}}$. Such an instanton cannot contribute to the superpotential unless the $\overline{\tau}^{\dot{\alpha}}$ modes are lifted by additional effects. This is what can happen e.g. for certain $E3$-instantons in Type IIB orientifolds in the presence of supersymmetric 3-form flux together with gauge flux on the instanton [17]. Having said this, for gauge instantons, i.e. if the instanton wraps a cycle parallel

to one of the spacetime-filling D-branes, the $\overline{\tau}^{\dot\alpha}$ act as Lagrange multipliers which enforce the fermionic ADHM constraints [32]. This effect was generalised to stringy instantons parallel to a single $U(1)$-brane in [22, 27].

In absence of such effects the instanton can nevertheless generate higher fermionic F-terms [17]. The difference to the F-terms generated by a non-rigid $O(1)$ instanton discussed previously is that now only the complex structure moduli receive derivative corrections. Denote by $w$ and $a$ the scalar and axionic parts of the scalar component $U = w - ia$ of a complex structure superfield. Then evaluation of the amplitudes $\langle\theta\,\overline{w}\,\overline{\tau}\rangle$ and $\langle\theta\,\overline{a}\,\overline{\tau}\rangle$ gives rise to the terms $\theta\,\sigma^\mu\,\overline{\tau}\,\partial_\mu\overline{w}^i$, $\theta\,\sigma^\mu\,\overline{\tau}\,\partial_\mu\overline{a}^i$, in the moduli action. The absence of analogous terms for the Kähler moduli is a consequence of $U(1)$ worldsheet charge conservation. Integrating out two copies thereof indeed generates a derivative coupling of the form $e^{-S_{E2}}\,\partial\overline{U}\partial\overline{U}$. Together with their fermionic partners, the derivative F-terms can be summarized by

$$S = \int d^4x\,d^2\theta\,\,e^{-\mathcal{U}(\Xi)}\,f_{\overline{i},\overline{j}}\left(e^{-\mathcal{T}_i}, e^{\Delta_i}\right)\,\overline{\mathcal{D}}^{\dot\alpha}\overline{\mathcal{U}}^{\overline{i}}\,\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{U}}^{\overline{j}} +\,\,h.c., \tag{8}$$

where the complex conjugate part is due to the anti-instanton contribution. In the presence of charged zero modes $\lambda$ these F-term corrections for the complex structure moduli involve appropriate powers of charged open string fields required to soak up the $\lambda$ modes. This amounts to replacing $e^{-\mathcal{U}(\Xi)} \to e^{-\mathcal{U}(\Xi)}\prod_i\Phi_{a_i,b_i}$.

To summarize we have shown that not only rigid but under certain circumstances also non-rigid $O(1)$-instantons give rise to superpotential contributions. The latter generically generate higher fermionic F-terms à la Beasley and Witten involving open and closed string superfields. Rigid $U(1)$-instantons with no intersections with its orientifold image induce derivative corrections to complex structure moduli space.

## 4   Instanton recombination

So far we have only considered instantons not intersecting their orientifold image, i.e those without uncharged zero modes arising from the $E2 - E2'$ sector. For more generic instantons exhibiting such zero modes to give contributions to the superpotential the fermionic zero modes arising at the intersections of $E2$ and $E2'$ need to be lifted in addition to the $\overline{\tau}^{\dot\alpha}$ modes [17]. A reason to expect that this is possible is the following: For $D6$-branes it is known that under certain circumstances a pair of $D6$-$D6'$ branes can recombine into a new sLag D6-brane which obviously wraps an $\Omega\overline{\sigma}$ invariant three-cycle. If a similar story also applies to pairs of $E2 - E2'$ instantonic branes, the recombined objects would be candidates for new $O(1)$-instantons contributing to the superpotential. Consequently, also the disjoint sum of $E2$ and $E2'$ prior to recombination should yield a superpotential contribution.

It turns out that an analysis of the $E2 - E2'$ and $\overline{\tau}$ - modes is only part of the story. Recall that generically an instanton intersects the present $D6$-branes, which gives rise to the previously described charged fermionic zero modes $\lambda$. From table 2 the overall $U(1)_E$ charge of these charged zero modes can be read off,

$$\sum_i Q_E(\lambda^i) = -\sum_a N_a\,\Xi \circ (\Pi_a + \Pi_{a'}) = -4\,\Xi \circ \Pi_{O6}. \tag{9}$$

Here we have used the tadpole cancellation condition. This shows that in a globally consistent model the total $U(1)_E$ charge of all charged zero modes is proportional to the chiral intersection between the instanton and the orientifold plane. For an $\Omega\overline{\sigma}$ invariant instanton this last quantity vanishes, whereas for a generic $U(1)$ instanton it does not.

If $\Xi \circ \Pi_{O6} \neq 0$, there must be additional $U(1)_E$-charged zero modes in order for the zero mode measure to be $U(1)_E$ invariant. These are the ones arising at the $E2 - E2'$ intersection and displayed in table 1.

### 4.1   Recombination of chiral $E2 - E2'$ instantons

In the following we assume that the instanton intersects its image exactly once on top of the orientifold,

$$\Xi' \circ \Xi = \Pi_{O6} \circ \Xi = 1. \tag{10}$$

As follows from the global consistency condition (9), the instanton suffers from an excess of additional four charged zero modes $\overline{\lambda}$. The $U(1)_E$ invariant zero mode measure reads [3]

$$\int d\mathcal{M}_I = \int d^4x\, d^2\theta d^2\overline{\tau}\, dm\, d\overline{m}\, \underbrace{d^2\overline{\mu}^{\dot\alpha}}_{Q_E=4}\, \underbrace{\prod_a d\lambda_a \prod_b d\overline{\lambda}_b}_{Q_E=-4}. \tag{11}$$

We first show that the uncharged modes in the $E2-E2'$ sector and the extra universal $\overline{\tau}$ can successfully be lifted upon taking into account the interaction of the instanton with its orientifold image [17]. The two crucial couplings in the instanton effective action are

$$S_{E2} = (2m\,\overline{m} - \xi)^2 + m\,\overline{\tau}_{\dot\alpha}\,\overline{\mu}^{\dot\alpha}, \tag{12}$$

where the Fayet-Iliopoulos $\xi$ depends on the complex structure moduli. It is proportional to the angle modulo 2 between the cycle $\Xi$ and its image $\Xi'$ and vanishes for supersymmetric configurations. For $\xi$ positive the bosonic modes $m$ become tachyonic and thus condense. This results in a new instanton wrapping a new cycle with homology class equal to $[\Xi] + [\Xi']$. The instanton computation is performed for $\xi = 0$, for which the CFT description of the effective action is valid. Integrating out two copies of the second term in (12) saturates the extra uncharged zero modes in the instanton measure. Upon performing the path integral over the bosonic modes this results in the measure

$$\int d\mathcal{M}_I = \int d^4x\, d^2\theta \underbrace{\prod_a d\lambda_a \prod_b d\overline{\lambda}_b}_{Q_E=-4}. \tag{13}$$

Although this measure has the correct uncharged zero mode structure to give rise to superpotential terms and thus looks quite encouraging, it turns out that there is no way to soak up the excess $\overline{\lambda}_b$ modes. Therefore the whole instanton amplitude vanishes and in contrast to the naive expectation the recombined $E2 - E2'$ instanton does not contribute to the superpotential.

### 4.2   Recombination of non-chiral $E2 - E2'$ instantons

In the previous section we have seen that once $E2 - E2'$ interactions are taken into acoount, even genuine $U(1)$ instantons effectively have the correct uncharged zero mode structure to give rise to the superpotential. However, the additional $\overline{\lambda}$ modes required to ensure global consistency spoil the game. Consequently, it may be more promising to consider an instanton intersecting non-chirally with its orientifold image such that no net excess of charged $\lambda$ modes is needed to satisfy (9).

The simplest non-trivial case involves one vector-like pair of zero modes, i.e.

$$[\Xi' \cap \Xi]^+ = [\Xi' \cap \Xi]^- = 1, \quad [\Pi_{O6} \cap \Xi]^+ = [\Pi_{O6} \cap \Xi]^- = 1. \tag{14}$$

It gives rise to the zero modes shown in table 3 and thus to the measure

$$\int d\mathcal{M}_I = \int d^4x\, d^2\theta d^2\overline{\tau}\, dm\, d\overline{m} dn\, d\overline{n}\, d^2\overline{\rho}^{\dot\alpha}\, d^2\overline{\nu}^{\dot\alpha} \underbrace{\prod_a d\lambda_a \prod_b d\overline{\lambda}_b}_{Q_E=0}. \tag{15}$$

Generically the instanton moduli action takes the form

$$S_{E2} = (2m\,\overline{m} - 2n\overline{n})^2 + \overline{\tau}_{\dot\alpha}(m\overline{\rho}^{\dot\alpha} - n\overline{\nu}^{\dot\alpha}). \tag{16}$$

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[3]  Note the inverse scaling behaviour of the Grassmann numbers.

| zero mode | $(Q_E)_{Q_{ws}}$ |
|-----------|------------------|
| $m, \overline{m}$ | $(2)_1$ ,$(-2)_{-1}$ |
| $\overline{\rho}^{\dot{\alpha}}$ | $(-2)_{1/2}$ |
| $n, \overline{n}$ | $(-2)_1$ ,$(2)_{-1}$ |
| $\overline{\nu}^{\dot{\alpha}}$ | $(2)_{1/2}$ |

**Table 3** Charged zero modes on non-chiral $E2 - E2'$ intersection with $O6^-$ plane.

Performing the integrals over the fermionic and bosonic zero modes one is left with the instanton measure

$$\int d\mathcal{M}_I = \int d^4x \, d^2\theta d^2\overline{\tau} \, dm \, d\overline{m} d^2\overline{\overline{\mu}}^{\dot{\alpha}} \underbrace{\prod_a d\lambda_a \prod_b d\overline{\lambda}_b}_{Q_E=0}, \tag{17}$$

where $\overline{\overline{\mu}}^{\dot{\alpha}}_{1/2} = \overline{\rho} + \overline{\nu}$. Note that this recombined instanton has precisely the same zero mode structure as an $O(1)$ instanton with one deformation mode discussed in section 3. There we showed that if particular couplings are present it can contribute to the usual superpotential F-terms [17]. Additionally it generates higher fermionic F-terms à la Beasley and Witten involving open and closed string superfields.

In certain situations there may be additional quartic couplings in the instanton effective action which allow one to integrate out also the net deformation modes $\overline{\overline{\mu}}^{\dot{\alpha}}_{1/2}$ [26]. In this case, the $E2 - E2'$ pair contributes to the superpotential also without invoking one of the mechanisms discussed in in section 3. Whether or not these terms are present can be read off uniquely from the dimension of the moduli space of the deformed cycle upon crossing the line of marginal stability. E.g. for non-chiral instanton recombination on toroidal orbifolds they are absent, as can be verified by a direct CFT computation.

## 5 Phenomenological applications

In this section we discuss two examples of instanton generated perturbatively forbidden superpotential terms in Type IIA orientifolds. As we will argue, the virtue of the instanton sector is not only to generate such terms in the first place. As a bonus the exponential suppression of the resulting operators with respect to the string scale allows for a natural generation of certain hierarchies which otherwise appear to be rather adhoc from a purely four-dimensional point of view. A most prominent example of such a hierarchical coupling is given by intermediate Majorana masses. Before analysing their generation by instanton effects we discuss an example of D-brane instanton generated Yukawa couplings in SU(5) GUT models.

### 5.1 $SU(5)$ Yukawa couplings

Grand Unified $SU(5)$-like models based on intersecting $D6$-branes generically suffer from the absence of the important Yukawa coupling $\mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}_H$ and are therefore ruled out from being considered realistic.

To construct such a model one only needs two stacks $a$ and $b$ of branes realising the gauge group $U(5)_a \times U(1)_b$. The $U(5)_a$ splits into $SU(5)_a \times U(1)_a$, where the anomalous $U(1)_a$ gets massive via the generalized Green-Schwarz mechanism and appears as a global symmetry in the effective action. While matter transforming as $\mathbf{10}$ under $SU(5)_a$ arises at intersections of stack $a$ with its image $a'$ the matter fields transforming as $\overline{\mathbf{5}}$ as well as Higgs fields $\mathbf{5_H}$ and $\overline{\mathbf{5}}_\mathbf{H}$ are located at intersections of stack $a$ with $b$ and $b'$. Assuming that after applying the generalized Green Schwarz mechanism only the combination $U(1)_X = \frac{1}{4}U_a - \frac{5}{4}U(1)_b$ remains massless one obtains a flipped $SU(5) \times U(1)_X$ model. Table 4 displays the abstract quiver structure of such a setup.

Looking at the $U(1)_{a,b}$ charges it is clear that perturbatively the two Yukawa couplings $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}^\mathbf{H}$ and $\overline{\mathbf{5}}\,\mathbf{5^H}\,\mathbf{1}$ are present. For flipped $SU(5)$ these gives masses to the heavy (u,c,t)-Quarks and the leptons.

| sector | number | $U(5)_a \times U(1)_b$ reps. | $U(1)_X$ |
|--------|--------|------------------------------|----------|
| $(a', a)$ | 3 | $\mathbf{10}_{(2,0)}$ | $\frac{1}{2}$ |
| $(a, b)$ | 3 | $\overline{\mathbf{5}}_{(-1,1)}$ | $-\frac{3}{2}$ |
| $(b', b)$ | 3 | $\mathbf{1}_{(0,-2)}$ | $\frac{5}{2}$ |
| $(a', b)$ | 1 | $\mathbf{5}^H_{(1,1)} + \overline{\mathbf{5}}^H_{(-1,-1)}$ | $(-1) + (1)$ |

**Table 4** GUT $SU(5)$ intersecting D6-brane model.

However, the Yukawa couplings for the light (d,s,b)-quarks

$$\mathbf{10}_{(2,0)}\,\mathbf{10}_{(2,0)}\,\mathbf{5}^H_{(1,1)} \tag{18}$$

are not invariant under the two $U(1)$s.

Now, if the model contains an $O(1)$-instanton with intersections [15]

$$[\Xi \cap \Pi_a]^+ = 0, \quad [\Xi \cap \Pi_a]^- = 1, \quad [\Xi \cap \Pi_b]^+ = 0, \quad [\Xi \cap \Pi_b]^- = 1, \tag{19}$$

then we get five zero modes $\overline{\lambda}^i_{[\overline{5}]}$ from the intersections of the instanton with $D6_a$ and one zero mode $\overline{\nu}_{[-1]}$ from the intersection with $D6_b$. Since the instanton lies in an $\Omega\overline{\sigma}$ invariant position, one can absorb these six matter zero modes with the three disc diagrams depicted in figure 1.
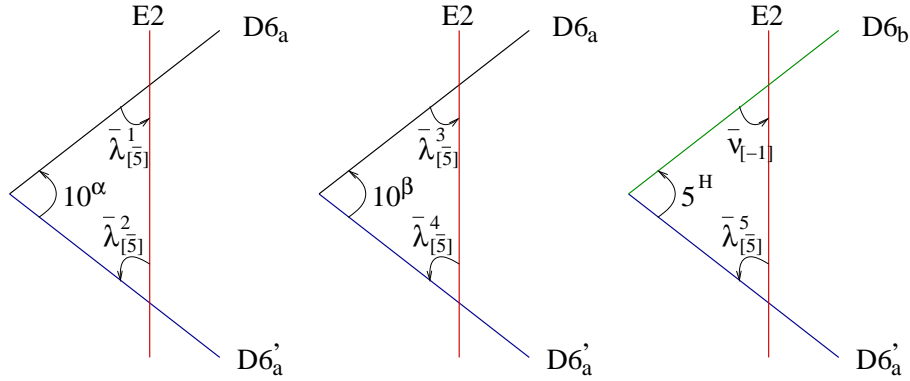


**Fig. 1** Absorption of zero modes.

All charge selection rules are satisfied. Due to the Grassmanian nature of the fermionic zero modes $\overline{\lambda}^i$ the index structure of the Yukawa coupling is

$$W_Y = x^{\alpha\beta}\, e^{-\frac{2\pi}{\alpha_{GUT}} \frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{D6}}}\, \epsilon_{ijklm}\, \mathbf{10}^{\alpha}_{ij}\, \mathbf{10}^{\beta}_{kl}\, \mathbf{5}^H_m, \tag{20}$$

where $\alpha, \beta = 1, 2, 3$ are generation indices. Here we have assumed that both D6-branes have the same volume and gauge coupling unification at the GUT scale. The parameter $x^{\alpha\beta}$ contains the contribution from the one-loop determinants and additional prefactors of order one [8]. The location of the matter fields depend on the flavour index and therefore also the Yukawa couplings do.

To summarize, we find that D-brane instantons can generate the $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ Yukawa coupling. For the case of flipped SU(5) the hierarchy between the $(u, c, t)$ quarks and the $(d, s, b)$ quarks is explained by the $E2$-instanton suppression $\exp(-S_{E2}(U))$, whereas the hierarchy between the first, second and third family is due to the different suppression of world-sheet disc instantons $\exp(-S_{ws}(T))$, where $U$ denote the complex structure and $T$ the Kähler superfields.

### 5.2   Majorana masses

Another interesting perturbatively forbidden coupling is the Majorana mass term for right-handed neutrinos. These Majorana masses together with non-perturbative Dirac masses give rise to the seesaw mechanism. Generically the right-handed neutrino is located at intersections between two abelian gauge groups $U(1)_a$ and $U(1)_b$ which become massive due to the generalized Green-Schwarz mechanism. In order to realise the remaining matter content of the MSSM we have to introduce additional branes. For simplicity let us focus on a supersymmetric $SU(5)$ model, where we need only one additional stack of 5 coincident branes providing an $SU(5)$ gauge group. Table 5 displays the matter content as well as the associated representations under the gauge groups.

| sector | number | $U(5)_a \times U(1)_b \times U(1)_c$ |
|--------|--------|--------------------------------------|
| $(a', a)$ | 3 | $\mathbf{10}_{(0,0)}$ |
| $(a, c)$ | 3 | $\overline{\mathbf{5}}_{(1,0)}$ |
| $(b, a)$ | 1 | $\mathbf{5}^H_{(1,0)} + \overline{\mathbf{5}}^H_{(-1,0)}$ |
| $(b, c)$ | 3 | $\mathbf{1}_{(-1,1)}$ |

**Table 5**  GUT $SU(5)$ intersecting D6-brane model.

We note that the coupling generating the Dirac mass is realized $\overline{\mathbf{5}}\,\mathbf{5}^H\,\mathbf{1}$ while a Majorana mass term for the righthanded neutrino

$$M_{N_R^c}\,\mathbf{1}_{(-1,1)}\,\mathbf{1}_{(-1,1)} \tag{21}$$

is forbidden due to the global $U(1)_b$ and $U(1)_c$. If the model contains an $O(1)$-instanton with intersections [1, 3, 8, 12, 14]

$$[\Xi \cap \Pi_b]^+ = 0, \quad [\Xi \cap \Pi_b]^- = 2, \quad [\Xi \cap \Pi_c]^+ = 2, \quad [\Xi \cap \Pi_c]^- = 0, \tag{22}$$

we get the right number of charged zero modes which can be absorbed via two disc diagrams. Performing the integration over the instantonic zero modes yields [1, 8]

$$M_{N_R^c}^{\alpha\,\beta} = x^{\alpha\,\beta}\,M_s e^{-\frac{2\pi}{\alpha_{GUT}}\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{D6_a}}}, \tag{23}$$

where $M_s$ is the string mass and $\alpha_{GUT}$ denotes the gauge coupling at the GUT scale. The prefactor $x$ receives contributions from the one-loop determinant as well as from the disc computation and is of order 1. Again the result is flavor dependent due to world sheet instantons. For a ratio of the volumes of $0.06 < \frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{D6_a}} < 0.02$ we obtain Majorana masses in the range of $10^{11} - 10^{15} GeV$ which yield, together with the perturbative Dirac neutrino masses, the desired hierarchically small see-saw masses. For a local supersymmetric realization as well as the computation of the instanton induced Majorana mass term for right-handed neutrino see [8].

These examples illustrate the importance of taking into account non-perturbative effects in exploring the string landscape. It is particularly desirable to discover globally defined realistic string vacua in which such and similar genuinely stringy effects naturally realize the hierarchies of scale which seem so puzzling from a purely four-dimensional effective field theoretical point of view. As we will see in the next section, such vacua can indeed be constructed in the framework of Tpye I compactifications.

## 6   $E1$-instantons in Type I compactifications

While the previous description of D-brane instantons is very illustrative in Type IIA language, Type IIB orientifolds are amenable to the techniques of complex geometry. This facilitates the construction of globally defined models. For this purpose we now consider the mirror symmetric formulation of stringy instantons

in Type I compactifications on an internal Calabi-Yau threefold $X$. The gauge sector is defined in terms of stacks of $M_a = n_a \times N_a$ spacetime-filling $D9$-branes wrapping the whole of $X$ (and their orientifold images), where $\sum_a n_a N_a = 16$. These $D9$-branes can carry rank $n_a$ holomorphic vector bundles $V_a$ whose structure group $U(n_a)$ breaks the original gauge group $U(M_a)$ associated with the coincident $D9$-branes to the commutant $U(N_a)$ [42, 43]. Stacks of $N_i$ $D5$-branes wrapping the holomorphic curve $\Gamma_i$ on $X$ carry gauge group $Sp(2N_i)$.

The massless open string spectrum is encoded in various cohomology groups associated with the respective bundles on the branes [42, 43]. Cancellation of $D5$-tadpoles requires $\sum_a N_a \mathrm{ch}_2(V_a) - \sum_i N_i \gamma_i = -c_2(TX)$, and absence of global anomalies is ensured by the constraint $\sum_a N_a c_1(V_a) \in H^2(X, 2\mathbf{Z})$.

The superpotential of the four-dimensional $\mathcal{N} = 1$ supersymmetric effective action receives non-perturbative corrections due to $E1$-instantons on a holomorphic curve $C$ [44]. For simplicity we focus on $E1$-branes wrapping rigid isolated $\mathbf{P}^1$s, which correspond on the Type IIA side to rigid $O(1)$-instantons. As described in the Type IIA context, in the presence of D-branes charged zero modes $\lambda$ arise, this time in the $D9 - E1$ sector. In the conventions of [24], fermionic zero modes in the representation $(N_a, 1_E)$ are counted by the cohomology group $H^0(C, V_a^\vee|_C \otimes K_C^{1/2})$, while the zero modes in the conjugate representation $(\overline{N}_a, 1_E)$ are associated with $H^0(C, V_a|_C \otimes K_C^{1/2}) = H^1(C, V_a^\vee|_C \otimes K_C^{1/2})^*$, see table 6.

| state | rep | cohomology |
|-------|-----|------------|
| $\lambda_a$ | $(N_a, 1_E)$ | $H^0(\mathbf{P}^1, V_a^\vee(-1)|_{\mathbf{P}^1})$ |
| $\overline{\lambda}_a$ | $(\overline{N}_a, 1_E)$ | $H^1(\mathbf{P}^1, V_a^\vee(-1)|_{\mathbf{P}^1})^*$ |

**Table 6** Fermionic zero modes in $D9 - E1$ sector.

These zero modes are particularly well under control if the $D9$-branes carry rank one, i.e. complex line bundles $L_a$. From the above it follows, with $K_{\mathbf{P}^1} = \mathcal{O}(-2)$, that the relevant object is $L_a(-1)|_{\mathbf{P}^1} = \mathcal{O}(x_a - 1)$, where $x_a = \int_{\mathbf{P}^1} L_a$. The charged zero modes are then readily determined by Bott's theorem. Additional fermionic zero modes from the $D5 - E1$ sector are counted by the extension groups $Ext_X(j_*\mathcal{O}|_{\Gamma_i}, i_*\mathcal{O}|_C)$. These groups vanish when $\Gamma_i$ and $C$ do not intersect.

If all $\lambda$ modes can be absorbed consistently the $E1$-instanton yields contributions to the superpotential of the schematic form

$$W = M_s^{3-k} \prod_{i=1}^{k} \Phi_i e^{-\frac{2\pi}{g_s} \frac{\mathcal{V}ol}{\ell_s^2}} = M_s^{3-k} \prod_i \Phi_i e^{-\frac{2\pi \ell_s^4}{\alpha_a} \frac{\mathcal{V}ol}{f_a}}. \tag{24}$$

Here we traded $g_s$ in for the gauge coupling on a reference brane $D9_a$. The scale of the non-perturbative term is thus controlled by the ratio of the instanton volume $\mathcal{V}ol = \int_{\mathbf{P}^1} J$ to the gauge kinetic function $\widetilde{f}_a$. This ratio is a function of the Kähler moduli which are in general only partially constrained by the D-flatness conditions. In (24) we also suppressed the possible dependence on the complex structure moduli through the one-loop Pfaffian [1, 6].

### 6.1 $E1$-instantons on elliptic CY3

We now present a class of globally defined supersymmetric models exhibiting such instanton effects. As Calabi-Yau threefold $X$ we choose a generic elliptic fibration $\pi : X \to B$ over the del Pezzo surface $B = dP_r$ for $r = 4$. The Kähler form $J$ of X enjoys the expansion $J/\ell_s^2 = r_\sigma \sigma + r_l \pi^* l + \sum_{i=1}^{4} r_i \pi^* E_i$ in terms of the fibre class $\sigma$ as well as the pullbacks of the the hyperplane class $l$ and the classes $E_i$ of the four $\mathbf{P}^1$s inside $dP_4$ obtained as the blow-up of certain singularities in $\mathbf{CP}^2$. The intersection form on $B$ is $l \cdot l = 1, E_i \cdot E_j = -\delta_{ij}$. An obvious class of rigid isolated $\mathbf{P}^1$ in $X$ consists in the ten horizontal $\mathbf{P}^1$s inherited from the base $B$, $E_i$ and $l - E_i - E_j, i \neq j$. They are described as the intersection of the zero section $\sigma$ with the divisors $\pi^* E_i$ or $\pi^*(l - E_i - E_j)$. Each $\pi^* E_i$ and $\pi^*(l - E_i - E_j)$ is itself an elliptic fibration over the respective horizontal $\mathbf{P}^1$ and thus represents a $dP_9$ surface. As such it contains an infinite

number of rigid isolated $\mathbf{P}^1$s, which are also rigid as curves in $X$. Consider for definiteness the divisor $\pi^* E_4$. Its second homology class $H_2(\pi^* E_4, \mathbf{Z})$ is spanned by the hyperplane class $h$ and the classes of the nine $\mathbf{P}^1$s $e_i, i = 1, \ldots, 9$ with intersection form $h \cdot h = 1, e_i \cdot e_j = -\delta_{ij}$ (see figure 2). However, these classes are not independent as elements in $H_2(X, \mathbf{Z})$, but are mapped under $\phi : H_2(\pi^* E_4, \mathbf{Z}) \to H_2(X, \mathbf{Z})$ to

$$\phi(e_i) = f + \sigma \pi^* E_4, \quad i = 1, \ldots 8, \quad \phi(e_9) = \sigma \pi^* E_4, \quad \phi(h) = 3(f + \sigma \pi^* E_4), \tag{25}$$

where $f$ denotes the fiber class as an element in $H_2(X, \mathbf{Z})$. In particular, we identify $e_9$, the base of the fibration $\pi^* E_4$, with the horizontal curve $\sigma \pi^* E_4$. The set of rigid isolated $\mathbf{P}^1$s in $\pi^* E_4$ is given by all integer combinations $d_0 h + \sum_i d_i e_i$ subject to $3d_0 + \sum_i d_i = 1 = \sum_i d_i^2 - d_0^2$ [45]. It follows that their class in $X$ is given by $(f + \sigma \pi^* E_4) + d_9 \sigma \pi^* E_4$. In the sequel we will be interested in the effect of $E1$-instantons wrapping one of these curves.
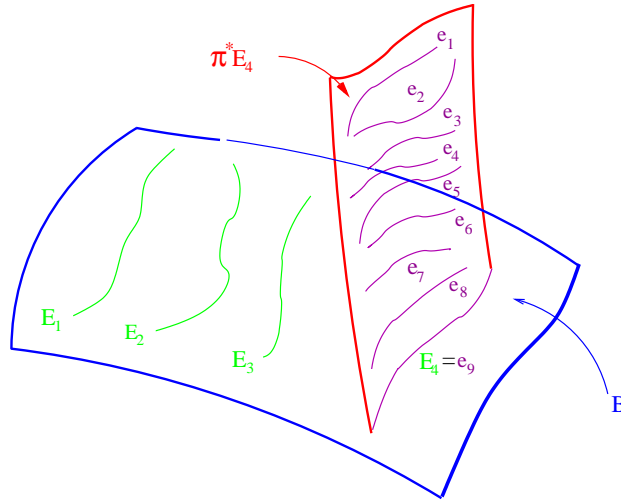


**Fig. 2**  The $dP_9$ surface $\pi^* E_4$ inside the firbration $\pi : X \to B$.

For our purposes it suffices to consider branes carrying line bundles $L_a$ with first Chern class $c_1(L_a) = q_a \sigma + \pi^* \zeta_a$ with $\zeta_a \in H^2(dP_4, \mathbf{Z})$. For the computation of charged zero modes of an instanton wrapping one of the above classes of curves it is important to note that $L_a|_{\mathbf{P}^1} = \mathcal{O}(\zeta_a \cdot E_4 - q d_9)$. Note furthermore that a D5-brane wrapping a $\mathbf{P}^1$ with $d_j = 0$ does not intersect the curve $e_j$ and therefore introduces no unwanted zero modes on an instanton wrapping $e_j$. On the other hand, those curves intersecting the D5-brane do carry additional zero modes and therefore yield different contributions to the effective action. This provides a way round the cancellation results of [46] for certain dual heterotic (0,2) non-linear $\sigma$-models on complete intersection manifolds.

### 6.2  Majorana masses

The described framework allows for the realisation of instanton generated superpotential couplings of various types in concrete, globally defined string vacua. This was demonstrated in [24] in the context of an SU(5) GUT model with instanton generated Majorana masses of the type described in section 5.2. In a second class of GUT models presented in [24], instantons generate a superpotential of Polonyi type in the hidden sector as a step towards hierarchically suppressed dynamical supersymmetry breaking.

As an illustration we consider the generation of Majorana masses. The GUT sector is located on a stack of $N_a = 5$ D9-branes endowed with line bundles $L_a$, while the right-handed neutrino $N_R^c$ transforms as $(-1_b, 1_c)$ of the gauge groups $U(1)_b$ and $U(1)_c$ realised as single D9-branes carrying line bundles $L_b$ and $L_c$. The zero mode constraints on the instanton for the generation of Majorana masses translate into

$h^i(\mathbf{P}^1, L_b^\vee(-1)|_{E_4}) = (2,0) = h^i(\mathbf{P}^1, L_c(-1)|_{E_4})$. For the subclass of $\mathbf{P}^1$s in $\pi^* E_4$ with $d_9 = 0$, this corresponds to $\zeta_b \cdot E_4 = -2 = -\zeta_c \cdot E_4$. The actual presence of the zero mode couplings $c_{ijk}\lambda_b^i (N_R^c)^j \overline{\lambda}_c^k$ in the moduli action is due to classical overlap integrals and we have checked that they indeed exist. Further details as well as the resulting family structure will be presented in a forthcoming publication [47].

In table 7 we give a representative example of an SU(5) model of the type described. All D5-brane tadpoles are cancelled by including also stacks of $N_i$ unmagnetized D5-branes on curves $\Gamma_i$ with total D5-brane charge $\sum N_i \gamma_i = 41F + \sigma \cdot \pi^*(16l - 12E_1)$, with $F$ the fibre class. Furthermore, 12 unmagnetized D9-branes are required to cancel the D9-brane tadpoles. One can check that the D-flatness conditions allow for solutions inside the Kähler cone, e.g., for $r_\sigma = 1.00, r_l = 10.39, r_1 = -7.00$ and $r_2 = r_3 = r_4 = -1.00$. The spectrum can be computed from the formulae of [42, 43]. It contains four chiral families

| Bundle | N | $c_1(L) = q\sigma + \pi^*(\zeta)$ |
|--------|---|-----------------------------------|
| $L_a$ | 5 | $\pi^*(-2E_3)$ |
| $L_b$ | 1 | $2\sigma + \pi^*(-2l - 2E_1 + 3E_2 + 2E_3 + 2E_4)$ |
| $L_c$ | 1 | $-2\sigma + \pi^*(2l - E_2 - 2E_3 - 2E_4)$ |

**Table 7** A $U(5) \times U(1) \times U(1)$ model with Majorana masses.

of $\overline{\mathbf{10}}$ counted by $H^*(X, (L_a^\vee)^2)$ together with additional $\mathbf{5}$ and $\overline{\mathbf{5}}$ from the $a - b$ and $a - c$ sector as well as from the filler D5- and D9-branes. Only part of them can be interpreted as matter $\mathbf{5}$ and Higgs pairs once Yukawa couplings are taken into account. Four chiral generations transforming as $(-1_b, 1_c)$ in the spectrum correspond to right-handed neutrinos $N_R^c$. The zero mode structure of the $E1$ instanton ensures that a superpotential of the form $W = x M_s \exp(-\frac{2\pi \ell_s^4}{\alpha_{GUT}} \frac{\mathcal{V}ol}{\widetilde{f}_a}) N_R^c N_R^c$ can be generated, where $x$ is an $\mathcal{O}(1)$ factor from the exact computation. The above value of the Kähler moduli corresponds to $\widetilde{f}_a = 9.56 \, \ell_s^6$ and an instanton volume $\mathcal{V}ol = -r_4 = 1$. Then for $M_s \sim 10^{18}$ GeV and $\alpha_{GUT} \sim 0.04$ the Majorana mass is $\mathcal{O}(10^{11})$ GeV. Thus, we demonstrated that in a global setup the exponential suppression can indeed be engineered at the required intermediate scale. The full answer involves summing up all instanton contributions associated with all suitable curves, but as alluded to already, depending on the concrete distribution of $D5$-branes required to cancel the tadpoles, it can be arranged that most of the curves do not contribute due to extra $D5 - E1$ zero modes. In favourable circumstances like the one given above it can even happen that only precisely one curve can contribute, in which case the associated superpotential term is guaranteed to be non-vanishing!

Finally, we note that the examples presented in [24] can be further refined, and indeed Standard-like models with 3 chiral generations of matter exist in this framework while still realising Majorana masses in the instanton sector. These and further aspects will be discussed in [47].

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